Road Network Repair Policy with Decentralized Control
分散制御に基づく道路ネットワークの補修施策

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When nearby road sections in a road network are repaired simultaneously, fixed repair costs, such as traffic regulation costs and installation costs of construction machines, can be reduced. However, when parallel road sections that are substitutes are repaired simultaneously, user cost may increase considerably because the traffic capacity of the entire road network may decrease significantly and become below the traffic demand. One effective method is to perform preventive repair, which means to repair road sections that have not reached their limit of use. The manager of a road network can change the combination of road sections repaired together by perform preventive repair to reduce fixed repair and user costs. In this study, we formulate a longterm management problem for road networks considering both repair and user costs. An approximate solution method based on decentralized control that can be applied to a largescale road network, where exact optimal repair policy is difficult to calculate, is also proposed. A numerical study in Sioux Falls network compared the proposed method and existing method. Compared with existing method, proposed method can reduce the annual repair cost by 60%, and reduce the annual user cost by 12.3%, and in total reduce the annual total social cost by 14.0%.

Key Words: *asset management, road repair policy, network-level optimization, economy of scale, preventive repair, dynamic programming, decentralized control*

1. Introduction

To guarantee the safety of a road network, the management needs to repair road sections that are severely damaged and have reached their limit of use. If multiple nearby road sections are repaired simultaneously, fixed repair costs, such as traffic regulation costs and installation costs of construction machines, can be reduced. Therefore, when repair works are performed nearby, preventive repair for road sections damaged but have not yet reached their limit of use can reduce longterm fixed repair costs, and hence, may reduce longterm total repair costs. The repair work of road sections may continue for several weeks and the traffic capacity of the sections being repaired is largely reduced due to the traffic constraint. Therefore, if the repair is performed simultaneously on parallel road sections that are substitutable for each other, the traffic capacity of the entire road network may be greatly reduced and below the traffic demand. In such a case, the users of the road will have to make a detour on a public road, which takes more time, thereby increasing the user cost. If parallel road sections will reach their limit of use simultaneously, preventive repair work on either road section can reduce the significant decrease in traffic capacity of the entire road network in near future.

In this study, we focus on the repair policy that can reduce the total longterm repair cost and user cost of a road network. The optimal repair policy that minimizes the total longterm cost is difficult to calculate because of the combination explosion when the size of a road network is large and therefore optimization methods from previous study only targeted on smallscale road network.[1)] Particularly, we propose an approximated repair policy based on decentralized control, which can be applied to a largescale road network. In the proposed method, the network is divided into multiple subnetworks comprising several neighboring road sections in advance by a static problem. As shown in **Fig. 1**, the proposed method divides the entire network into two subnetworks, and all road sections in either subnetwork are widely repaired. Compared with the independent repair policy of repairing only road sections that have reached their limit of use, nearby road sections are repaired simultaneously, and parallel road sections are repaired at different times. The numerical study applied proposed method to a real-scale road network and showed that the repair policy by the proposed method can reduce both repair cost and user cost than independent repair policy that do not perform preventive repair.

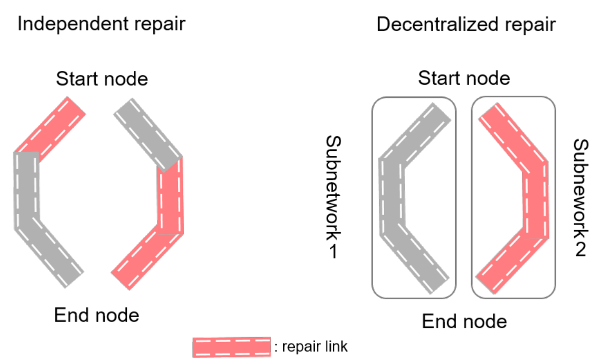


Fig. 1 Independent repair and decentralized repair

2. Road network model

(1) Deterioration and repair of road network

Consider a road network comprising L road links, denoted as $(i, j) \in \mathcal{L}$, where the start node is i and the end node is j. The condition state (CS) of link $(i, j) \in \mathcal{L}$ is defined as $s_{i,j}$ and is a discrete integer variable ranging from 1 to M. $\mathcal{M} \equiv \{1, 2, \cdots, M\}$ is the state space of CS. CS 1 means the best condition, and the condition worsen as CS increases; CS M means the worst condition. The condition of the entire road network can be denoted by CS vector $\boldsymbol{s}$, which is an L dimension vector:

$$\boldsymbol{s} \equiv [s_{i,j}], \quad \forall (i, j) \in \mathcal{L}. \tag{1}$$

The state space of CS vector can be obtained via a direct product of CS of all links and can be denoted as $\mathcal{S} = \mathcal{M}^{|\mathcal{L}|}$.

In this study, we do not consider the effect of traffic volume on deterioration and assume that deterioration only depends on time. Therefore, we assume that the deterioration of each link is denoted by the Markov deterioration process. If link (i, j) is not repaired at the zth time of inspection and repair t_z, the probability that the CS of link (i, j) changed from a to b is given by:

$$p_{a,b} = \mathrm{Prob}[s_{i,j}(t_{z+1}) = b | s_{i,j}(t_z) = a], \tag{2}$$
$$\forall z \in \mathbb{Z}^+, \quad \forall a, b \in \mathcal{M}.$$

The deterioration of link (i, j) canbe denoted by Markov transition matrix $\boldsymbol{P}$:

$$\boldsymbol{P} \equiv [p_{a,b}], \quad \forall a, b \in \mathcal{M}. \tag{3}$$

In terms of the entire network, the probability that the CS vector changes from $\boldsymbol{s}^*$ to $\boldsymbol{s}^{**}$ when no repair conducted can be denoted as follows:

$$\mathrm{Prob}[\boldsymbol{s}(t_{z+1}) = \boldsymbol{s}^{**} | \boldsymbol{s}(t_z) = \boldsymbol{s}^*] \tag{4}$$
$$= \prod_{(i,j)\in\mathcal{A}} p_{s^*_{i,j} s^{**}_{i,j}}, \quad \forall z \in \mathbb{Z}^+.$$

When a link is repaired, its condition becomes better and the CS decreases. In this study, we consider only one repair method that reduces the CS of the link repaired to 1 and the repair work takes time span d. At time $t \in \mathcal{T}$, repair on the entire network can be denoted by a binary vector $\boldsymbol{\delta}(t_z) \equiv \{\delta_{i,j}(t_z) | \forall (i, j) \in \mathcal{A}\}$. $\delta_{i,j}(t_z) = 1$ means repair is performed on link (i, j) from t_z to t_{Z+1}, $\delta_{i,j}(t_z) = 1$ means no repair is performed on link (i, j) and $\delta_{i,j}(t_z) = 0$ means no repair performed on link (i, j) from t_z to t_{Z+1}.

(2) Formation of the social cost of the road network

In this subsection, we formulate the construction cost incurred by the repair work. The construction cost incurred in the repair work comprises two types of costs: variable repair costs which means material and labor costs proportional to the length of the link to be repaired, and fixed repair costs which means costs of transporting construction equipment and traffic control costs. Fixed repair costs are assumed to occur only once when repair work is performed on adjacent links. Here adjacent links mean multiple links connected to the same node. The length of link (i, j), given by the variable repair cost proportional to the length of the link, is α, and the fixed repair cost proportional to the number of related nodes is β. The construction cost $r(\boldsymbol{\delta}(t_z)$ occurs when the repair vector is $\boldsymbol{\delta}(t_z)$ is defined as follows:

$$r(\boldsymbol{\delta}(t_z)) = \alpha \sum_{(i,j)\in\mathcal{L}} l_{i,j}\delta_{i,j} + \beta \sum_{n\in\mathcal{N}} b_n, \tag{5}$$

where $\quad \delta_{i,j}(t) \in \{0, 1\} \quad \forall (i, j) \in \mathcal{L}, \tag{6}$

$$b_i = \begin{cases} 0 & \text{if} \quad \delta_{i,j} = 0 \quad \text{and} \quad \delta_{j,i} = 0, \\ 1 & \text{else}, \end{cases}$$
$$\forall (i, j) \in \mathcal{L}, b_n \in \{0, 1\} \quad \forall n \in \mathcal{N}. \tag{7}$$

where b_n is a binary variable that indicates whether a node is related to a repair.

Here is the formulation of the user cost of the road network according to the repair. We assume that the traffic demand of the road network is always constant, independent of the state of the network, traffic regulations, and time of day. If there are K types of traffic demand in the network, the traffic demand set $\mathcal{F}$ representing them is denoted as $\mathcal{F} \equiv \{(O_k, D_k, f_k) | \forall k \in \mathcal{K}\}, \mathcal{K} \equiv \{1, 2, \cdots, K\}$. Here, O_k is the starting point of the traffic demand of the kth traffic demand, D_k is the ending point of the traffic demand of the kth traffic demand, and f_k is the quantity of the traffic demand of the kth traffic demand. The user cost of 1-unit flow on link $(i, j) \in \mathcal{L}$ for one period d is $c_{i,j}$, the capacity of flow on link $(i, j) \in \mathcal{L}$ is $\mu_{i,j}$, and the amount of traffic of the kth traffic demand on link $(i, j) \in \mathcal{L}$ from time t_z to t_{z+1} is $y_{i,j,k}(t_{z+1})$. On the link where the repair is conducted from t_z to t_{z+1}, the traffic capacity will be $\gamma (0 \le \gamma \le 1)$ times until t_{z+1}. If the traffic demand exceeds the network capacity, users will have to make a detour on an alternative road with high transport costs, and the cost of using the alternative road is significantly higher than the cost of using the targeted road network. In this study, the user cost for the alternative road is expressed as C_{O_k,D_k} and the traffic volume is expressed as $y^*_{O_k,D_k,k}(t_z)$. The user cost is a minimum cost flow problem, and when the repair vector is $\boldsymbol{\delta}(t_z)$, the user cost $u(\boldsymbol{\delta}(t_z))$ incurred between time t_z and t_{Z+1} is defined as follows:

$$u(\boldsymbol{\delta}(t_z)) = \min_{\boldsymbol{y}(t_z), \boldsymbol{y}^*(t_z)} \sum_{k\in\mathcal{K}} \sum_{(i,j)\in\mathcal{L}} c_{i,j} y_{i,j,k}(t_z) + C_{O_k,D_k} y^*_{O_k,D_k,k}(t_z), \tag{8}$$

$$\text{s.t.} \quad \sum_{(n,j)\in\mathcal{L}} y_{n,j,k}(t_z) - \sum_{(i,n)\in\mathcal{L}} y_{i,n,k}(t_z) = f_k, \tag{9}$$
$$\forall n \in \mathcal{N} \setminus \{O_k, D_k\}, \quad \forall k \in \mathcal{K},$$
$$\sum_{(O_k,n)\in\mathcal{L}} y_{O_k,n,k}(t_z) + y^*_{O_k,D_k,k}(t_z) - \sum_{(n,O_k)\in\mathcal{L}} y_{n,O_k,k}(t_z)$$
$$= f_k, \quad \forall n \in \{O_k | \forall k \in \mathcal{K}\}, \quad \forall k \in \mathcal{K}, \tag{10}$$
$$\sum_{(n,D_k)\in\mathcal{L}} y_{n,D_k,k}(t_z) + y^*_{O_k,D_k,k}(t_z) - \sum_{(n,D_k)\in\mathcal{L}} y_{n,D_k,k},(t_z)$$
$$= f_K, \quad \forall n \in \{D_k | \forall k \in \mathcal{K}\}, \quad \forall k \in \mathcal{K}, \tag{11}$$
$$\sum_{k\in\mathcal{K}} y_{i,j,k}(t_z) \le \mu_{i,j}(1 - \rho\delta_{i,j}(t_z)), \quad \forall (i, j) \in \mathcal{L}, \tag{12}$$
$$y_{i,j,k}(t_z) \ge 0, \quad \forall (i, j) \in \mathcal{L}, \quad \forall k \in \mathcal{K}, \tag{13}$$
$$y^*_{O_k,D_k,k}(t_z) \ge 0, \quad \forall \{O_k, D_k, k\} \in \mathcal{F}. \tag{14}$$

3. Repair policy

(1) Life-cycle costs of the road network

In this study, repair measure is method(s) to determine the decision of links to repair based on the current information. We assume that the management must perform repair work on the link(s) with the maximum CS, CS M. In other words, the following holds for the repair vector $\boldsymbol{\delta}(t_z, d)$, which means CS vectors determined by the measure d:

$$\delta_{i,j}(t, d) = 1, \quad \text{if} \quad s_{i,j}(t) = 1, \quad \forall (i, j) \in \mathcal{L}, \quad \forall t \in \mathcal{T}. \tag{15}$$

The life-cycle cost (LCC) is the total long-term social cost of a newly constructed road network from time t_0. If ρ is the discount rate of time cost, the LCC of the road network under the repair measure d applied can be calculated as

Table. 1 Determination of subnetwork(subNW.) to conduct preventive repair by proposed method

		Num. of max CS in subNW. 2	
		$< X_2$	$\geq X_2$
Num. of max CS in subNW. 1	$< X_1$	SubNW. 1	SubNW.2
	$\geq X_1$	SubNW. 1	SubNW.1

follows:

$$LCC(d, t_0) = E\Big[\sum_{t \in \mathcal{T}} \rho^t [r(\boldsymbol{\delta}^d(t)) + u(\boldsymbol{\delta}^d(t))]\Big]. \tag{16}$$

The independent repair policy, which does not consider user cost and repair cost of road networks and only perform repair(s) on link(s) with the maximum CS, can be expressed as follows:

$$\boldsymbol{\delta}^{d^0}(t) = [\delta^{d^0}_{i,j}(t)], \tag{17}$$

$$\text{where} \quad \delta^{d^0}_{i,j}(t) = \begin{cases} 1 & \text{if} \quad s_{i,j} = M. \\ 0 & \text{else.} \end{cases} \tag{18}$$

(2) Exact solution: The optimal repair policy

The optimal repair policy d^* that minimizes the long-term discounted user cost and repair work cost for the entire network is defined as follows:

$$d^* = \arg\min_d LCC(d, t_0). \tag{19}$$

The formulated problem is a Markov process, the optimal repair policy comprises a repair vector that depends only on the current state, not on the past state or time. Thus, the determination of the repair vector by the optimal repair strategy d^* is defined as follows:

$$\boldsymbol{\delta}^{d^*} = f^{d^*}(\boldsymbol{s}). \tag{20}$$

Smallscale optimal repair policies can be calculated using algorithms such as the value iteration method and policy iteration method[2)], but it is impossible to calculate the optimal repair policy for a practical-scale road network due to the combinatorial explosion. Optimal repair policies for a road network comprising more than 10 links is hard to calculate.

(3) Approximate solution: Repair policy with decentralized control

In this study, we propose an approximate solution method with decentralized control and variable approximation. In the proposed method, the road network is first divided into multiple subnetworks comprising neighboring links. By only performing preventive repairs on any subnetwork, it is possible to prevent a significant decrease in the traffic capacity of the entire road network due to the simultaneous construction of subnetworks, and by performing largescale preventive repairs within any subnetwork, it is possible to reduce the repair work cost by repairing neighboring road links. The subnetworks are decided by a shortterm problem that does not consider the stochastic deterioration process. The shortterm problem is a cost minimization problem for a network of period T. The problem assumes that each link in the network does not deteriorate and only needs to be repaired once in the period T, and it can be formulated as follows:

$$\min_{\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \cdots, \boldsymbol{\delta}_{T^*}} \sum_{t=1}^{T} r(\boldsymbol{\delta}_t) + u(\boldsymbol{\delta}_t), \tag{21}$$

$$\text{s.t.} \quad \sum_{t=1}^{T} \boldsymbol{\delta}_t = \mathbf{1}. \tag{22}$$

where $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \cdots, \boldsymbol{\delta}_T$ are the repair vectors at each time, which divide the links to be repaired simultaneously and

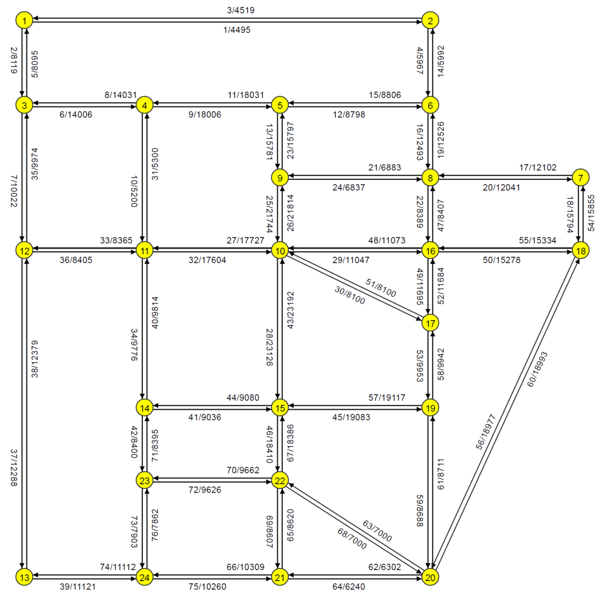


Fig. 2 Sioux Falls network used in numerical study

Table. 2 Parameters used in numerical study

Traffic demand set: $\mathcal{F}$	$\{(1, 20, 2, 500), (2, 13, 2, 500)\}$			
Traffic volume decrease rate by traffic constraint: γ	1			
Cost of public road: C	300			
Variable repair cost: α	10			
Fixed repair cost: β	20			
Discount rate of time to cost: ρ	0.95			
Preventive repair boarder: $\hat{M}$	3			
Span of short-term problem: T	2			
Markov transition probability: $\boldsymbol{P}$	0.512	0.365	0.106	0.018
	0	0.578	0.333	0.089
	0	0	0.638	0.362
	0	0	0	1

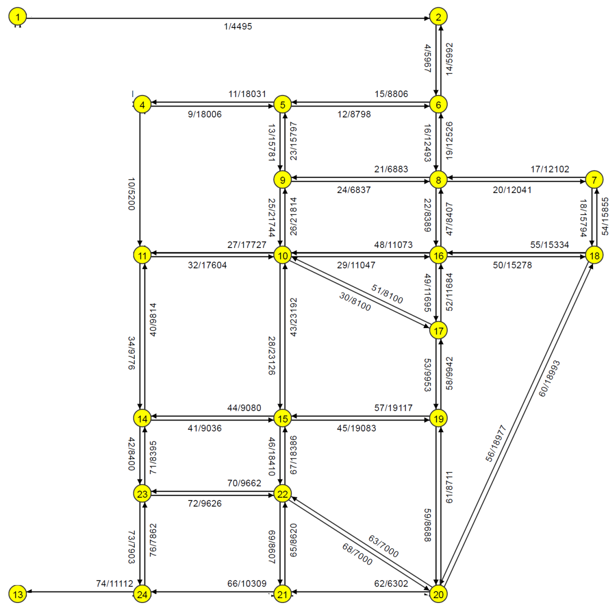
the nodes containing those links into the same subnetwork. By this process, the entire network is divided into multiple subnetworks such that the increase in the total social cost due to simultaneous construction in a single subnetwork is small. The divided subnetworks are denoted as $\mathcal{G}_1(\mathcal{N}_1, \mathcal{L}_1), \mathcal{G}_2(\mathcal{N}_2, \mathcal{L}_2), \cdots, \mathcal{G}_D(\mathcal{N}_D, \mathcal{L}_D)$.

Next is the method for determining the subnetworks to have preventive repair. At each time $t \in \mathcal{T}$, the subnetwork for preventive repair is determined according to the condition of all subnetworks. The state space of all subnetworks is reduced by approximating the CS vectors of the subnetworks with a binary variable, "Is the number of links with the maximum deterioration state greater than or equal to the borderline X of each subnetwork?" The subnetwork to repair is determined by the approximated condition of each subnetwork; **Table.1** is an example when the number of subnetworks is 2. In the subnetwork where preventive repair is performed, the repair vector is denoted as follows:

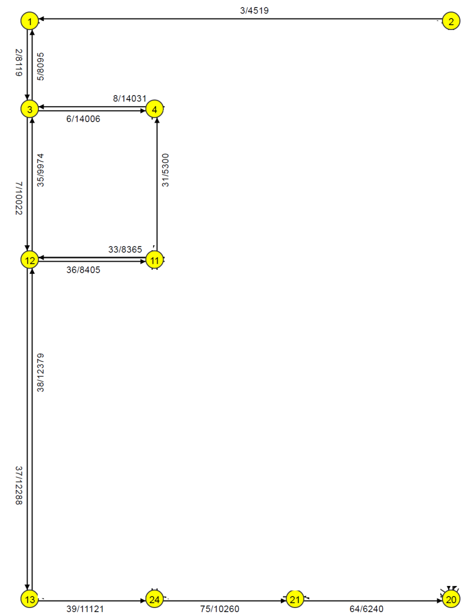
$$\boldsymbol{\delta}^{d^0}(t) = [\delta^{d^0}_{i,j}(t)], \tag{23}$$

$$\text{where} \quad \delta^{d^0}_{i,j}(t) = \begin{cases} 1 & \text{if} \quad s_{i,j} \geq \hat{M}. \\ 0 & \text{else.} \end{cases} \tag{24}$$

Preventive repair boarder $\hat{M}(\hat{M} < M)$ is a given variable (e.g. $\hat{M} = M - 1$). Optimal borderlines and subnetworks to repair are decided by simulation for all combinations of all borderlines and subnetworks to repair.



(a) SubNW.1



(b) SubNW.2

Fig. 3 Subnetwroks by proposed method

Table. 3 Determination of subnetwork to perform preventive repair using the proposed method in a numerical study

		Num. of max CS in subNW. 2	
		< 2	≥ 2
Num. of max CS in subNW. 1	< 9	SubNW. 1	SubNW.2
	≥ 9	SubNW. 1	SubNW.1

Table. 4 Annual costs by two policies

	Independent repair	Proposed method
Repair cost	118,63	4,740
User cost	202,051	179,184
Social cost	213,914	183,924
Alternative road traffic flow	314.38	52.72

4. Numerical study

(1) Settings of numerical study

In the numerical study, the Sioux Falls network **Fig.2** was used to compare i) independent repair that only repairs links with maximum CS, and ii) the proposed repair policy by the approximate solution method. The numerical study assumes that traffic volume decrease rate by traffic constraint $\gamma = 1$, which means link being repaired can not be used. The precise parameters used in numerical study are shown in **Table.2**.

(2) Result of numerical study

Fig.3 shows two subnetworks divided by the proposed method and we can find that each subnetwork contains paths for both traffic demand. Subnetwork 1 comprised of 61 links and subnetwork 2 comprised of 15 links. **Table.3** shows the subnetwork to perform preventive repair according to the approximated condition of all subnetworks. **Table.4** shows the annual repair cost, the annual user cost, the annual social cost, and the annual alternative road traffic flow. Compared with independent repair, annual repair cost is reduced to 40.0%; annual user cost is reduced to 88.7%; annual total cost is reduced to 86.0%. In this case, the repair policy by proposed method can reduce repair cost and user cost simultaneously and therefore is more desirable than the independent repair policy.

5. Summary

In this study, we propose a methodology for deriving optimal repair policies at the network level, considering the uncertainty of the deterioration process. Specifically, the user cost of the repair work is calculated considering the substitutability and complementarity of each link in a road network, and a repair policy problem is formulated to obtain the optimal repair policy considering the economies of scale of the user and repair costs. We also proposed an approximation solution method with decentralized control to avoid combinatorial explosion. In the proposed methodology, the network to be analyzed was divided into subnetworks based on the shortterm problem. At every decision-making time, the subnetwork to perform preventive repair is decided based on the combination of the approximated conditions of all subnetworks. The optimal parameters for proposed methodology are calculated by simulation. In the numerical study, the proposed method was applied to a real-scale case study to verify its effectiveness. Compared with existing methodology that do not consider repair policies at the network level, the proposed methodology can reduce both repair and user costs in long term.

Future studies may focus on multiple road network with substitutability. In this study, we focused on user costs by overcapacity of a single network. However, the highway network and the public road network may exist in the same city. Management considering repair work, detour, and user cost of both network may lead to a more practical management for the urban transportation network.

Reference

1) Lethanh, N., Adey, B. T. and Burkhalter, M.: Determining an optimal set of work zones on large infrastructure networks in a GIS framework, *Journal of Infrastructure Systems*, Vol.24, Issue 1, 04017048, 2017.
2) Bellman, R.: A Markovian decision process, *Journal of Mathematics and Mechanics*, Vol.6, No.5, pp.679-684, 1957.

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